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ASTRONOMICAL PROBLEM.

Having the times when a circumpolar ſtar paſſed the thread of a teleſcope, above and below the pole, to determine the azimuth of the teleſcope, and the times when the ſtar paſſed the meridian.

By THEOPHILUS PARSONS, Eſq. A.A.S.

HAVING obſerved by a clock, the times at which a circumpolar ſtar paſſed the perpendicular thread of a tranſit teleſcope, both above and below the pole, to determine from thence, the azimuth of the teleſcope, and the times at which the ſtar paſſed the meridian, both above and below the pole.

Subtract the time when the ſtar paſſed the thread at the firſt obſervation, from the time it paſſed the thread at the ſecond obſervation, firſt adding twelve hours to the latter time, and if the remainder, or interval between the two paſſages thus found, is equal to the time employed by the ſtar, meaſured by the ſame clock, in making half a revolution, then the teleſcope is placed in the meridian.

If that interval be leſs than the time of half a revolution, the teleſcope has a weſtern azimuth, but if that interval be greater, it has an eaſtern azimuth; that is, when the firſt obſervation was made above the pole; but when the firſt obſervation is made under the pole, the contrary is to be obſerved. Now to determine this azimuth, obſerve the following

RULE.

RULE.

Having turned into parts of a circle, half the difference between that interval, and half of a revolution of the ſtar, = half the ſum of the errors in the times of the tranſit, make tang. *'s pol. diſt. × ſine $\frac{1}{2}$ diff. = tang. of an arc, which call X : then ſine X × ſec. lat. = ſine azimuth required.

To determine the times when the ſtar paſſed the meridian, both above and below the pole, obſerve the following

RULE.

Let tang. *'s pol. diſt. × ſine $\frac{1}{2}$ diff. × tang. lat. = to the ſine of an arc, which call Y : turn this arc into time, and the ſum of this arc, and the half difference aforeſaid, is the error in time, of the tranſit below the pole ; and the difference between this arc, and the ſaid half difference, is the error in time of the tranſit above the pole,

Or more briefly, the half ~~difference, in time abovementioned~~, is equal to half the ſum of the errors in the times of the tranſit, both above and below the pole, and the arc Y, turned into time, is equal to half the difference of the ſaid errors.

The error in the tranſit above the pole, is to be added to the time of the tranſit, when the teleſcope has an eaſtern azimuth, and is to be ſubtracted, when it has a weſtern azimuth ; but the error in the tranſit below the pole, is to be contrarily applied.

When the teleſcope is nearly in the meridian, ſo that its error in azimuth does not amount to a degree, the rules may be made more compendious, by uſing logiſtic logarithms.

1. For the error in azimuth.

L L. ½ diff. + log. tang. ∗'s declin. + log. coſin. lat. = L. log. of the error in azimuth.

2. For half the difference in time of the errors of the tranſit, above and below the pole.

L. log. ½ diff. + log. tang. ∗'s declin. + log. cotang. lat. = L. log. of half the difference in time, of the errors required.

EXAMPLE I.

In latitude 42° 48′ 6″ N. *a* Urſ. maj. was obſerved to paſs the teleſcope above the pole, at 6^h 32′ 4″ in the morning, and to paſs the ſame teleſcope in the evening, at 5^h 22′ 56″ under the pole, by a clock which kept mean ſolar time: required the poſition of the teleſcope, and the times when the ſtar paſſed the meridian, above and below the pole.

	h	′	″
Time of the tranſit below the pole, + 12^h	17	22	56
Time of the tranſit above the pole,	6	32	4
Interval,	10	50	52
Half rev. of the ſtar by that clock,	11	58	2
Difference,	1	7	10
Half difference = half ſum of the errors,		33	35
	°	′	″
Turned into parts of a circle, at the rate of 23^h 56′ 4″ : 360°	8	25	8
N. dec. of *a* Urſ. maj.	62	54	8

Tang.

Tang. of ſtar's pol. diſt.	27°	5′	52″	9·7089959
Sine of half diff.	8	25	8	9.1655683
Tang. of arc X,	4	17	3,4	8.8745642
Sine of arc X,	4	17	3,4	8.8733504
Sec. of lat.	42	48	6	10.1344755
Sine of azimuth weſt,	5	50	37,9	9·0078259

Now to find the times of the tranſit.

Tang. of ſtar's pol. diſt.	27°	5′	52″	9·7089959
Sine of ½ diff. = ½ ſum of errors,	8	25	8	9.1655683
Tang. of lat.	42	48	6	9·9666410
Sine of arc Y = ½ diff. of errors,	3	58	41	8.8412052

	h	′	″
The arc Y turned into time, at the rate of 360° : 23ʰ 56′ 4″		15	52
Half the diff. in time above found,		33	35
Their diff. is error of the tranſit above the pole,		17	43
Their ſum, is error of do. below the pole,		49	27
Time of tranſit above the pole,	6	32	4
Error minus,		17	43
Time of paſſing the meridian above the pole,	6	14	21
Time of tranſit below the pole,	5	22	56
Error plus,		49	27
Time of paſſing the meridian below the pole,	6	12	23

PROOF.

PROOF.

From the time of paſſing the merid. below the pole,	6	12	23
Subtract the time of paſſing do. above the pole,	6	14	21
There remains (= to half the time of the ſtar's revolution by the clock)	11	58	2

EXAMPLE II.

Suppoſe every thing as in the laſt example, only that the ſtar

paſſed above the pole in the morning, at	6^{h}	$0'$	$0''$
And below it in the evening, at	6	3	56
Interval,	12	3	56
Half revolution of the ſtar,	11	58	2
Difference in time,		5	54
Half diff. = half ſum of the errors,		2	57
The ſame in parts of a circle,	0°	$44'$	$22''$

1. To find the azimuth.

L. log. of ½ diff. = ½ ſum of errors,	0°	$44'$	$22''$	0.1311
Log. tang. of ∗'s declination,	62	54	8	10,2909
Log. coſine of latitude,	42	48	6	9.8655
L. log. error in azimuth, eaſt,	0	30	57	0.2875

2. The errors of the tranſit.

L. log. of ½ diff. = ½ ſum of errors,	0°	$44'$	$22''$	0.1311
Log. tang. of ∗'s declination,	62	54	8	10.2909
Log. cotang. of latitude,	42	48	6	10.0333
L. log. of ½ diff. of errors = arc Y,	0	21	2	0.4553

$21'$

	h	′	″
21′ 2″ turned into time, is	0	1	24
Half diff. above found, is		2	57
Error above the pole,	0	1	33
Error below the pole,	0	4	21
The time of tranſit above the pole,	6	0	0
Error plus,		1	33
Time of paſſing the meridian above the pole,	6	1	33
Time of tranſit below the pole,	6	3	56
Error minus,		4	21
Time of paſſing the meridian below the pole,	5	59	35

PROOF.

Time of paſſing the meridian below the pole,	5	59	35
Subtract time of paſſing do. above the pole,	6	1	33
Remains (= ½ a ſtar's revolution,)	11	58	2

DEMONSTRATION.

IN the annexed Figure, (Plate I. fig. 3.) let the primitive circle, W Y E, repreſent the plane of the horizon, W Æ E the equator, ZPY the meridian, P the north pole, Z the zenith, BCDF the parallel deſcribed by the circumpolar ſtar, Z A the poſition of the teleſcope, interſecting the parallel of the ſtar in C and B, and then the angle A Z Y will be the azimuth of the teleſcope. Draw the horary circles P C and P B, continuing the latter to F. Then the arc B C is deſcribed in the interval between the obſervations, (ſuppoſing the firſt obſervation made above the pole) and the angle B P C is known from the time elapſed during the interval. The arc B C F is half of the parallel of the ſtar, and is deſcribed by it, in half a revolution. The angle B P Y = D P F is the error in the time of the tranſit below the pole, and the angle C P D is the error in the time of the tranſit above the pole. The angle C P F, the ſum of thoſe errors, is the ſupplement to the angle B P C. Let fall the perpendicular P N, and it will biſect the angle B P C, becauſe B P and C P are equal. Draw P x biſecting the angle C P F, then becauſe C P F is the ſupplement of the angle B P C, C P x the half of C P F is the complement of the angle C P N, the half of B P C; and conſequently the angle N P x is a right angle: therefore the angle D P x is the complement of the angle N P Z. Now becauſe C P x (= x P F) is equal to half of the ſum of the errors, in the times of the tranſits, D P x is equal to half of the difference of thoſe errors; for C P x (= x P F) + D P x = D P F the greateſt error, and C P x — D P x = C P D the leaſt error.

The formulas now to be demonſtrated, are to find the angle N Z P the azimuth, and the angle D P x the half difference of the errors in the times of the tranſits, called in the ſecond formula the arc Y.

1. To find the angle N Z P.

In the triangle N P C, are given the ſide P C, the ſtar's polar diſtance, and the angle N P C, known by the half interval of time elapſed between the obſervations, (or rather its complement C P x, the half ſum of the errors in the times of the tranſits,

ſits, known by turning into parts of a circle, half of the difference of the time between the interval of the two paſſages, and the time of half the ſtar's revolution) to find the ſide N P, which in the firſt formula is called the arc X.

By ſpherics we have Rad : tang. P C :: coſine N P C (= ſine C P x) : tang. N P. Therefore tang. N P = ſine C P x × tang. P C.

Again, in the triangle N P Z: we have N P juſt found, and the ſide Z P the colatitude of the place of obſervation, to find the angle N Z P the azimuth required.

Sine Z P (= coſine P Y the lat.): Rad :: ſine N P : ſine N Z P, the azimuth, then ſine N Z P $= \frac{\text{ſine N P } (=X)}{\text{coſine P Y}} =$ ſine X × ſecant P Y.

Q. E. D.

When logiſtic logarithms are to be uſed, and the azimuth is ſmall, the ſine and tangent of N P do not ſenſibly differ from each other, nor from the arc N P; and the arc C P x does not ſenſibly differ from its ſine.

If therefore we ſubſtitute ſine C P x × tang. P C, for ſine N P, the formula is reduced to ſine N Z P = ſine C P x × tang. P C × ſecant P Y. But if we uſe the logiſtic logarithm of the angle C P x, we muſt take the complements of the tangent P C, and the ſecant P Y, which gives us this compendious formula. L. log. of the azimuth = L. log. half the ſum of the errors turned into parts of a circle + log. tang. of the ſtar's declination + log. coſine of the latitude of the place.

Q. E. D.

Q. E. D.

2. To find the angle D P x (= arc Y.)

Having already tang. N P = ſine C P x × tang. P C, in the triangle N P Z are given N P and P Z to find the angle N P Z, or its complement D P x, the angle required.

Rad: tang. N P (= ſine C P x × tang. P C) : : Cotang. P Z (= tang. P Y) : coſine N P Z (= ſine D P x.) Therefore ſine D P x = ſine C P x × tang. P C × tang. P Y. And by uſing the logiſtic logarithms of the angles D P x, and C P x, and the complements of the tangents P C and P Y, we have this formula. L. log. of half the difference of errors in parts of a circle = L. log. of ½ ſum of the errors in parts of a circle + log. tang. of ſtar's declination + log. cotang. of the lat. of the place.

Q. E. D.

N. B. If the clock goes at a different rate from that mentioned in the foregoing examples, proper allowance muſt be made in determining the angles, from the times of obſervation.

November, 1786.

Remarks on an Eclipſe of the Moon ſeen at Jeruſalem, not long before the death of Herod, and mentioned by Joſephus. By JAMES WINTHROP, Eſq. A.A.S.

THE eclipſe, which is the ſubject of the preſent paper, is of great importance in chronology, as it ſerves to aſcertain the true point of time from which the Chriſtian æra ought to be